

An Adjoint Separation Logic for the Wasm Call Stack

Work in Progress!

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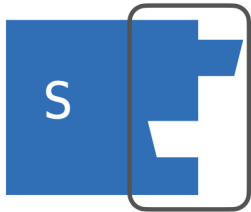
Context: Formally Specifying ABIs

Application Binary Interface (ABI)

The run-time contract for using a particular API

– Swift

Context: Formally Specifying ABIs



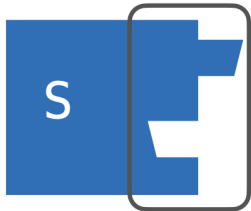
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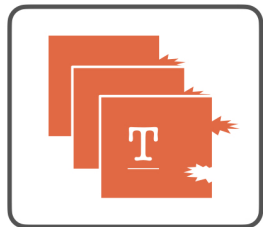


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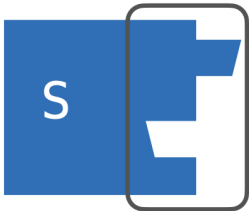


Is *Realistically Realized* [Benton06]

By These Target Programs

$\llbracket \tau \rrbracket = \{ \underline{e} \mid \dots \}$

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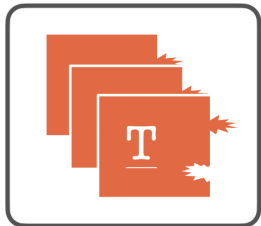


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e is **ABI compliant** with τ if

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OOPSLA24

Realistic Realizability: Specifying ABIs You Can Count On

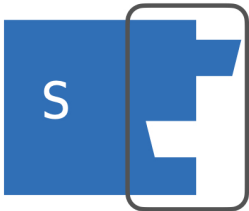
ANDREW WAGNER, Northeastern University, USA

ZACHARY EISBACH, Northeastern University, USA

AMAL AHMED, Northeastern University, USA

The Application Binary Interface (ABI) for a language defines the interoperability rules for its target platforms, including data layout and calling conventions, such that compliance with the rules ensures “safe” execution and perhaps certain resource usage guarantees. These rules are relied upon by compilers, libraries, and foreign-function interfaces. Unfortunately, ABIs are typically specified in prose, and while type systems for source languages have evolved, ABIs have remained largely unchanged, leaving those systems in a state of uncertainty and safety.

Context: Formally Specifying ABIs



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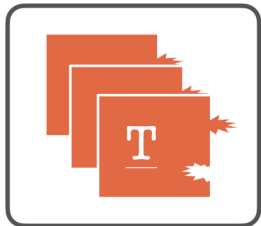
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Target-level separation
logic predicate

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Realistic Realizability: Specifying ABIs You Can Count On

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Toward a Rust ABI for Wasm



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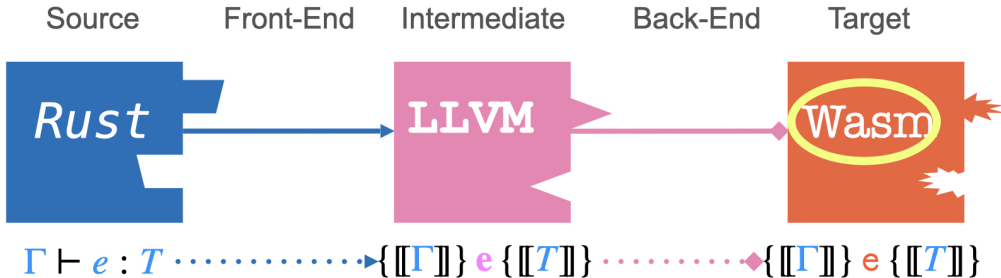
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Toward a Rust ABI for Wasm



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1. A **semantics** for borrowing
2. To support *independent updates* to the **front-end** and **back-end**
3. A **separation logic** for Wasm

State of the Art: IrisWasm

PLDI23



Iris-Wasm: Robust and Modular Verification of WebAssembly Programs

XIAOJIA RAO*, Imperial College London, UK
AÍNA LINN GEORGES*, Aarhus University, Denmark
MAXIME LEGOUPIL†, Aarhus University, Denmark
CONRAD WATT, University of Cambridge, UK
JEAN PICHON-PHARABOD, Aarhus University, Denmark
PHILIPPA GARDNER‡, Imperial College London, UK
LARS BIRKEDAL‡, Aarhus University, Denmark

WebAssembly makes it possible to run C/C++ applications on the web. A WebAssembly program is expressed as a collection of higher-order modules, which are put together through a system of explicit imports and exports using a higher-order modular programming. We present Iris-Wasm, a mechanized framework for reasoning on a specification of Wasm 1.0 mechanized in Coq and the Iris framework. We specify and verify individual modules separately, and then compose them to verify the whole program.

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Iris-MSWasm: Elucidating and Mechanising the Security Invariants of Memory-Safe WebAssembly

MAXIME LEGOUPIL, Aarhus University, Denmark
JUNE ROUSSEAU, Aarhus University, Denmark
AÍNA LINN GEORGES, MPI-SWS, Germany
JEAN PICHON-PHARABOD, Aarhus University, Denmark
LARS BIRKEDAL, Aarhus University, Denmark

WebAssembly offers coarse-grained encapsulation guarantees via its module system, but does not support fine-grained sharing of its linear memory. MSWasm is a recent proposal which extends WebAssembly with fine-grained memory management and sharing. We present Iris-MSWasm, a mechanized framework for reasoning about the security invariants of MSWasm. We specify and verify individual modules separately, and then compose them to verify the whole program.

IrisWasm: Sample wp Rules

$$\begin{array}{c}
 \text{wp_binop} \\
 \frac{\llbracket t.\text{binop} \rrbracket(c_1, c_2) = c * \triangleright \Phi(\text{immV } [t.\text{const } c]) * \xrightarrow{\text{FR}} F}{\text{wp } [t.\text{const } c_1; t.\text{const } c_2; t.\text{binop } \text{binop}] \{w, \Phi(w) * \xrightarrow{\text{FR}} F\}}
 \\
 \text{wp_call} \\
 \frac{(F.\text{inst.funcs}[i] = \text{addri}) * \xrightarrow{\text{FR}} F * \triangleright \left(\xrightarrow{\text{FR}} F \multimap \text{wp } [\text{invoke } \text{addri}] \{w, \Phi(w)\} \right)}{\text{wp } [\text{call } i] \{w, \Phi(w)\}}
 \\
 \text{wp_invoke_native} \\
 \frac{\begin{array}{l} |vs| = |ts_I| * cl = \{(inst; ts); es\}_{(ts_I \rightarrow ts_2)}^{\text{NativeCl}} * F' = \{\text{locs} := vs \mathrel{++} \text{zeros}(ts); \text{inst} := inst\} * \\ i \xrightarrow{\text{wf}} cl * \xrightarrow{\text{FR}} F * \triangleright \left[\begin{array}{c} (i \xrightarrow{\text{wf}} cl * \xrightarrow{\text{FR}} F) \multimap \\ \text{wp } [\text{local}_{|ts_2|}\{F'\} \text{ (block } ([\] \rightarrow ts_2) \text{ es) end}] \{w, \Phi(w)\} \end{array} \right] \end{array}}{\text{wp } (vs \mathrel{++} \text{invoke } i) \{w, \Phi(w)\}}
 \\
 \text{wp_local_bind} \\
 \frac{\xrightarrow{\text{FR}} F * \left(\xrightarrow{\text{FR}} F_1 \multimap \text{wp } es \left\{ w, \exists F'_1, \xrightarrow{\text{FR}} F'_1 * \left(\xrightarrow{\text{FR}} F \multimap \text{wp } [\text{local}_n\{F'_1\} \text{ w end}] \{w', \Phi(w')\} \right) \right\} \right)}{\text{wp } [\text{local}_n\{F_1\} \text{ es end}] \{w', \Phi(w')\}}
 \end{array}$$

What do all of these rules have in common?

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 \end{array}
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What do all of these rules have in common?

Explicit threading of the monolithic “frame resource” $\hookrightarrow_{\text{FR}} F$

Context Splitting

$$\begin{array}{c} \Gamma \vdash e : T \\ \Downarrow \\ \{[\Gamma]\} \textcolor{red}{e}^* \{[T]\} \end{array}$$

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$$\begin{array}{c} \Gamma \vdash e : T \qquad (\Gamma = \overline{x : T_x}) \\ \Downarrow \\ \{F \star \dots\} e^* \{ \llbracket T \rrbracket \} \quad (F = \overline{\$x \mapsto v_x}) \end{array}$$

Context Splitting

$$\begin{array}{c} \otimes I \\ \frac{\Gamma_1 \vdash e_1 : T_1 \quad \Gamma_2 \vdash e_2 : T_2}{\Gamma_1, \Gamma_2 \vdash (e_1, e_2) : T_1 \otimes T_2} \\ \Downarrow \\ \{F_1 \star F_2 \star \dots\} \quad e_1^*; e_2^* \quad \{[T_1] \star [T_2]\} \end{array}$$

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Local points-to: $x \xrightarrow{\text{loc}} v$

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Rules are small footprint:

WP-PLUS

$$\frac{\Phi(\text{i32.const } (n_1 + n_2))}{\text{wp } (\text{i32.const } n_1; \text{i32.const } n_2; \text{i32.add}) \{ \Phi \}}$$

WP-LOC-GET

$$\frac{\$x \xrightarrow{\text{loc}} v \star \Phi(v)}{\text{wp } (\text{local.get } \$x) \{ \Phi \}}$$

WP-LOC-SET

$$\frac{\$x \xrightarrow{\text{loc}} v' \star (\$x \xrightarrow{\text{loc}} v \multimap \Phi(\epsilon))}{\text{wp } (v; \text{local.get } \$x) \{ \Phi \}}$$

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WP-FRAME-BIND[†]

$$\frac{F \multimap \text{wp } (e^*) \{ v^n. \Phi(v^n) \}}{\text{wp } (\text{frame}_n F e^*) \{ \Phi \}}$$

Problem 1: Popped Frames Not Encapsulated

$$\frac{}{\text{wp } (\text{frame}_0 (\text{locals } (\$x \ 42))) \ \epsilon \ \{ \$x \xrightarrow{\text{loc}} 42 \}}$$

Local $\$x$ should not be returnable from the `frame`. This *should* be false.

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$$\frac{\overline{\$x \xrightarrow{\text{loc}} 42 \rightarrow^* \mathbf{wp} (\epsilon) \{ \$x \xrightarrow{\text{loc}} 42 \}}}{\mathbf{wp} (\text{frame}_0 (\text{locals} (\$x \ 42)) \ \epsilon) \{ \$x \xrightarrow{\text{loc}} 42 \}} \text{WP-FRAME-BIND}^\dagger$$

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Binding under a frame moves the postcondition under a pop:

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$$\frac{}{\$x \xrightarrow{\text{loc}} 42 \vdash \text{wp} (\text{frame}_1 (\text{locals}) \text{local.get } \$x) \{v. v = \text{i32.const } 42\}}$$

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Adjoint Logic

Benton and Wadler (1996), Reed (2009)

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Push and pop are both modalities:

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They form an adjunction:

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$$\frac{\updownarrow\text{-UNIT}}{P \vdash \uparrow \downarrow P}$$

$$\frac{\downarrow \uparrow\text{-CUNIT}}{\downarrow \uparrow P \vdash P}$$

Example: Suspending and Resuming Frames

$$\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp } ((\text{frame}_1 (\text{locals}) \text{i32.const } 25) ; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}$$

Example: Suspending and Resuming Frames

$$\begin{array}{c} \$x \stackrel{\text{loc}}{\mapsto} 20 \vdash \text{wp} \left(\text{frame}_1 (\text{locals}) \text{i32.const } 25 \right) \{v'. \text{wp} (v'; \text{local.get } \$x; \text{i32.add}) \{v.v = 45\}\} \\ \text{---(WP-CTX-BIND)---} \\ \$x \stackrel{\text{loc}}{\mapsto} 20 \vdash \text{wp} \left((\text{frame}_1 (\text{locals}) \text{i32.const } 25) ; \text{local.get } \$x; \text{i32.add} \right) \{v. v = 45\} \end{array}$$

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$$\begin{array}{l} \textcolor{brown}{\downarrow}(\$x \xrightarrow{\text{loc}} 20) \vdash \text{wp} \text{ (i32.const 25) } \{v'. \downarrow \text{wp} (v'; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}\} \\ \text{---}(\uparrow \text{ -ADJ})\text{---} \\ \$x \xrightarrow{\text{loc}} 20 \vdash \uparrow \text{wp} \text{ (i32.const 25) } \{v'. \downarrow \text{wp} (v'; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}\} \\ \text{---}(\text{WP-FRAME-BIND})\text{---} \\ \$x \xrightarrow{\text{loc}} 20 \vdash \text{wp} \text{ (frame}_1 \text{ (locals) i32.const 25) } \{v'. \text{wp} (v'; \text{local.get } \$x; \text{i32.add}) \{v.v = 45\}\} \\ \text{---}(\text{WP-CTX-BIND})\text{---} \\ \$x \xrightarrow{\text{loc}} 20 \vdash \text{wp} \text{ ((frame}_1 \text{ (locals) i32.const 25) ; local.get } \$x; \text{i32.add) } \{v. v = 45\} \end{array}$$

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—(\downarrow -MONO)—

$\downarrow (\$x \xrightarrow{\text{loc}} 20) \vdash \downarrow \text{wp} \text{ (i32.const 25; local.get } \$x; \text{i32.add)} \{v. v = 45\}$

—(WP-VAL)—

$\downarrow (\$x \xrightarrow{\text{loc}} 20) \vdash \text{wp} \text{ (i32.const 25)} \{v'. \downarrow \text{wp} (v'; \text{local.get } \$x; \text{i32.add)} \{v. v = 45\}\}$

—(\uparrow -ADJ)—

$\$x \xrightarrow{\text{loc}} 20 \vdash \uparrow \text{wp} \text{ (i32.const 25)} \{v'. \downarrow \text{wp} (v'; \text{local.get } \$x; \text{i32.add)} \{v. v = 45\}\}$

—(WP-FRAME-BIND)—

$\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp} \text{ (frame}_1 \text{ (locals) i32.const 25)} \{v'. \text{wp} (v'; \text{local.get } \$x; \text{i32.add)} \{v.v = 45\}\}$

—(WP-CTX-BIND)—

$\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp} \text{ ((frame}_1 \text{ (locals) i32.const 25); local.get } \$x; \text{i32.add)} \{v. v = 45\}$

Example: Suspending and Resuming Frames

$$\begin{array}{l} \text{---(WP-CTX-BIND, WP-LOCAL-GET, WP-BINOP, WP-VAL)---} \\ \$x \xrightarrow{\text{loc}} 20 \vdash \text{wp } (\text{i32.const } 25; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\} \\ \text{---}(\downarrow \text{-MONO})\text{---} \\ \downarrow (\$x \xrightarrow{\text{loc}} 20) \vdash \downarrow \text{wp } (\text{i32.const } 25; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\} \\ \text{---(WP-VAL)---} \\ \downarrow (\$x \xrightarrow{\text{loc}} 20) \vdash \text{wp } (\text{i32.const } 25) \{v'. \downarrow \text{wp } (v'; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}\} \\ \text{---}(\uparrow \text{-ADJ})\text{---} \\ \$x \xrightarrow{\text{loc}} 20 \vdash \uparrow \text{wp } (\text{i32.const } 25) \{v'. \downarrow \text{wp } (v'; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}\} \\ \text{---(WP-FRAME-BIND)---} \\ \$x \xrightarrow{\text{loc}} 20 \vdash \text{wp } (\text{frame}_1 (\text{locals}) \text{i32.const } 25) \{v'. \text{wp } (v'; \text{local.get } \$x; \text{i32.add}) \{v.v = 45\}\} \\ \text{---(WP-CTX-BIND)---} \\ \$x \xrightarrow{\text{loc}} 20 \vdash \text{wp } ((\text{frame}_1 (\text{locals}) \text{i32.const } 25); \text{local.get } \$x; \text{i32.add}) \{v. v = 45\} \end{array}$$

Other Stacks

Other Stacks

A shadow stack

$$\begin{array}{c} \text{WP-SPUSH} \\ \frac{\uparrow^s \Phi(\epsilon)}{\text{wp } (\$push) \{ \Phi \}} \\ \hline \end{array} \quad \begin{array}{c} \text{WP-SPOP} \\ \frac{\downarrow_s \Phi(\epsilon)}{\text{wp } (\$pop) \{ \Phi \}} \\ \hline \end{array} \quad \begin{array}{c} \text{WP-SALLOC} \\ \frac{\forall n', b^n. n' \xrightarrow{\text{stk}} b^n \rightarrow \star \Phi(\text{i32.const } n')}{\text{wp } (\$salloc \ n) \{ \Phi \}} \\ \hline \end{array}$$

Other Stacks

A shadow stack

$$\begin{array}{c}
 \text{WP-SPUSH} \\
 \frac{\uparrow^s \Phi(\epsilon)}{\text{wp } (\$push) \{ \Phi \}} \\
 \\
 \text{WP-SPOP} \\
 \frac{\downarrow_s \Phi(\epsilon)}{\text{wp } (\$pop) \{ \Phi \}} \\
 \\
 \text{WP-SALLOC} \\
 \frac{\forall n', b^n. n' \xrightarrow{\text{stk}} b^n \rightarrow \star \Phi(\text{i32.const } n')}{\text{wp } (\$salloc \ n) \{ \Phi \}}
 \end{array}$$

The operand stack?

$$\text{top}(20) \star \downarrow_o \text{top}(25) \star \downarrow_o \downarrow_o \text{top}(10) \approx \text{i32.const } 10; v; \text{i32.const } 25; \text{i32.const } 20$$

Other Stacks

A shadow stack

$$\begin{array}{c} \text{WP-SPUSH} \\ \frac{\uparrow^s \Phi(\epsilon)}{\text{wp } (\$push) \{ \Phi \}} \end{array} \quad \begin{array}{c} \text{WP-SPOP} \\ \frac{\downarrow_s \Phi(\epsilon)}{\text{wp } (\$pop) \{ \Phi \}} \end{array} \quad \begin{array}{c} \text{WP-SALLOC} \\ \frac{\forall n', b^n. n' \xrightarrow{\text{stk}} b^n \rightarrow \star \Phi(\text{i32.const } n')}{\text{wp } (\$salloc \ n) \{ \Phi \}} \end{array}$$

The operand stack?

$$\text{top}(20) \star \downarrow_o \text{top}(25) \star \downarrow_o \downarrow_o \text{top}(10) \approx \text{i32.const } 10; v; \text{i32.const } 25; \text{i32.const } 20$$

Stack switching?

Summary

WP-FRAME-BIND

$$\frac{\uparrow (F \rightarrow^* \text{wp} (e^*) \{v^n. \downarrow \Phi(v^n)\})}{\text{wp} (\text{frame}_n F e^*) \{\Phi\}}$$

$\uparrow\downarrow\text{-ADJ}$

$$\frac{\downarrow P \vdash Q}{P \vdash \uparrow Q}$$

